Euler systems

Winter term 2023/24

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The material of each lecture below should be carefully and completely explained even if this takes more than 90 minutes. All references are to [Rub00] if not indicated otherwise.

1) Local cohomology groups. Definition 1.1.1 and 1.1.3, and Sections 1.3. and 1.4 completely. Just state Theorem 1.4.1 as we assume all duality results for Galois cohomology to be known.

2) Selmer groups. Sections 1.5, 1.6. without Example 1.6B, and Section 1.7.

3) Euler systems and main results. Sections 2.1. (without Remarks 2.1.3 and 2.1.5), Section 2.2. (treat the hypotheses carefully), Section 2.3. and Section 2.4.

4) Example: cyclotomic units. Section 3.2. Give an idea of the proof of Corollary 3.2.4.

5) Example: elliptic curves with CM I. Review the theory of elliptic curves with CM after [Rub99] Section 5, especially Theorem 5.11, Theorem 5.15 and its corollaries 5.16-5.22. For further details it is useful to consult [Sil94] Chapter II.

6) Example: elliptic curves with CM II. Explain the construction of elliptic units from [Rub99] Section 7 and give the definition of the Euler system Def. 8.1 and Prop. 8.2. in loc. cit. and conclude with Theorems 3.3.1 and 3.3.2 in [Rub00].

7) The derivative construction. Sections 4.1, 4.2.,4.3. and 4.4.

8) Local properties of derivative classes. Sections 4.5., 4.6. and 4.7.

9) Bounding the order of the Selmer group. Explain Sections 5.1 and 5.2. and explain the necessary modifications in the proof of Theorem 2.2.3 from Section 5.3.

10) Twisting of Euler systems. Sections 6.1, 6.2, 6.3, 6.4 and 6.5.

11) Iwasawa theory I. Sections 7.1, 7.2 and 7.3.

12) Iwasawa theory II. Sections 7.4., 7.5., 7.6 and 7.7.

References

[Rub99] Karl Rubin, *Elliptic curves with complex multiplication and the conjecture of Birch and Swinnerton-Dyer*, Arithmetic theory of elliptic curves (Cetraro, 1997), Lecture Notes in Math., vol. 1716, Springer, Berlin, 1999, pp. 167–234. MR 1754688

[Rub00] ———, *Euler systems*, Annals of Mathematics Studies, vol. 147, Princeton University Press, Princeton, NJ, 2000, Hermann Weyl Lectures. The Institute for Advanced Study. MR 1749177

[Sil94] Joseph H. Silverman, *Advanced topics in the arithmetic of elliptic curves*, Graduate Texts in Mathematics, vol. 151, Springer-Verlag, New York, 1994. MR 1312368